TERRAMETRA

INEQUALITIES

Terrametra Resources

Lynn Patten

1.7
INEQUALITIES

- Linear Inequalities
- Three-Part Inequalities
- Quadratic Inequalities
- Rational Inequalities

PROPERTIES of INEQUALITY

PROPERTIES of INEQUALITY

Let a, b and c represent real numbers.

1. If $a < b$, then $a + c < b + c$.
2. If $a < b$ and if $c > 0$, then $ac < bc$.
3. If $a < b$ and if $c < 0$, then $ac > bc$.

Replacing $<$ with $>$, $\leq$, or $\geq$ results in similar properties.

PROPERTIES of INEQUALITY

Note

Multiplication may be replaced by division
in Properties 2 and 3.

Always remember to reverse the direction of the inequality symbol when multiplying or dividing by a negative number.

Linear Inequalities

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LINEAR INEQUALITY in ONE VARIABLE

A ***linear inequality in one variable***
is an inequality that can be written in the form
$$\boldsymbol{ax + b < 0},$$
Where $\boldsymbol{a}$ and $\boldsymbol{b}$ are real numbers, with $\boldsymbol{a \neq 0}$.
(Any of the symbols $>$, $\leq$, and $\geq$ may also be used.)

Example 1
Solving a Linear Inequality

1(a) Solve: $-3x + 5 > -7$

Solution:

$$-3x + 5 - 5 > -7 - 5$$ Subtract 5.

$$-3x > -12$$ Combine like terms.

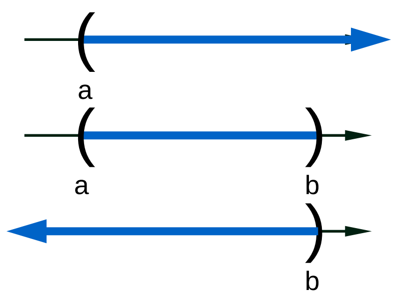
$$\frac{-3x}{-3} < \frac{-12}{-3}$$ Divide by -3.

Reverse the direction of the inequality symbol when multiplying or dividing by a negative number.

Don't forget to reverse the symbol here.

$$x < 4$$

TYPES of INTERVALS
INTERVAL NOTATION

TYPE OF INTERVAL	SET	INTERVAL NOTATION	GRAPH
All Real Numbers	$\{x \mid x \text{ is a real number}\}$	$(-\infty, \infty)$	
Open Intervals	$\{x \mid x > a\}$ $\{x \mid a < x < b\}$ $\{x \mid x < b\}$	(a, ∞) (a, b) $(-\infty, b)$	
Closed Interval	$\{x \mid a \leq x \leq b\}$	$[a, b]$	

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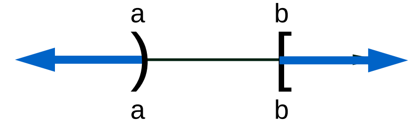
TYPES of INTERVALS
INTERVAL NOTATION

TYPE OF INTERVAL	SET	INTERVAL NOTATION	GRAPH
Other Intervals	$\{x \mid x \geq a\}$	$[a, \infty)$	
	$\{x \mid a \leq x < b\}$	$[a, b)$	
	$\{x \mid a < x \leq b\}$	$(a, b]$	
	$\{x \mid x \leq b\}$	$(-\infty, b]$	
Disjoint Intervals	$\{x \mid x < a \text{ or } x > b\}$	$(-\infty, a) \cup (b, \infty)$	
	$\{x \mid x < a \text{ or } x \geq b\}$	$(-\infty, a) \cup [b, \infty)$	
	$\{x \mid x \leq a \text{ or } x > b\}$	$(-\infty, a] \cup (b, \infty)$	
	$\{x \mid x \leq a \text{ or } x \geq b\}$	$(-\infty, a] \cup [b, \infty)$	

Example 2

Solving a Linear Inequality

2(a) Solve: $4 - 3x \leq 7 + 2x$

Give the solution set in interval notation.

Solution:

$4 - 3x \leq 7 + 2x$	
$4 - 3x - 4 \leq 7 + 2x - 4$	Subtract 4.
$-3x \leq 3 + 2x$	Combine like terms.
$3x - 2x \leq 3 + 2x - 2x$	Subtract $2x$.
$-5x \leq 3$	Combine like terms.

Example 2
Solving a Linear Inequality

2(a) Solve: $4 - 3x \leq 7 + 2x$

Give the solution set in interval notation.

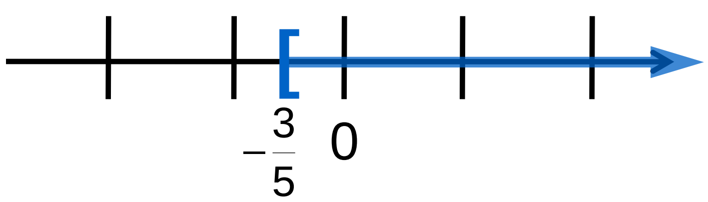
Solution (cont'd):

$$-5x \leq 3$$

$$\frac{-5x}{-5} \geq \frac{3}{-5}$$

Divide by -5.
Reverse the direction of the inequality symbol.

$$x \geq -\frac{3}{5}$$



In interval notation the solution set is $\left[-\frac{3}{5}, \infty\right)$

Example 3

Solving a Three-Part Inequality

3(a) Solve: $-2 < 5 + 3x < 20$

Solution:

$$-2 - 5 < 5 + 3x - 5 < 20 - 5$$ Subtract 5. (*from each part*)

$$-7 < 3x < 15$$ Combine like terms.

$$\frac{-7}{3} < \frac{3x}{3} < \frac{15}{3}$$ Divide *each part* by 3.

$$-\frac{7}{3} < x < 5$$ Simplify.

The solution set is $\left(-\frac{7}{3}, 5\right)$

Quadratic Inequalities

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QUADRATIC INEQUALITIES

QUADRATIC INEQUALITIES

A ***quadratic inequality***

is an inequality that can be written in the form

$$ax^2 + bx + c < 0$$

for real numbers a, b, and c, with $a \neq 0$.

(The symbol $<$ can be replaced with $>$, $\leq$, or $\geq$.)

Solving a Quadratic Inequality

Solving a Quadratic Inequality

Step 1 Solve the corresponding quadratic equation.

Step 2 Identify the intervals determined by the solutions of the equation.

Step 3 Use a test value from each interval to determine which intervals form the solution set.

Example 4
Solving a Quadratic Inequality

4(a) Solve: $x^2 - x - 12 < 0$

Solution:

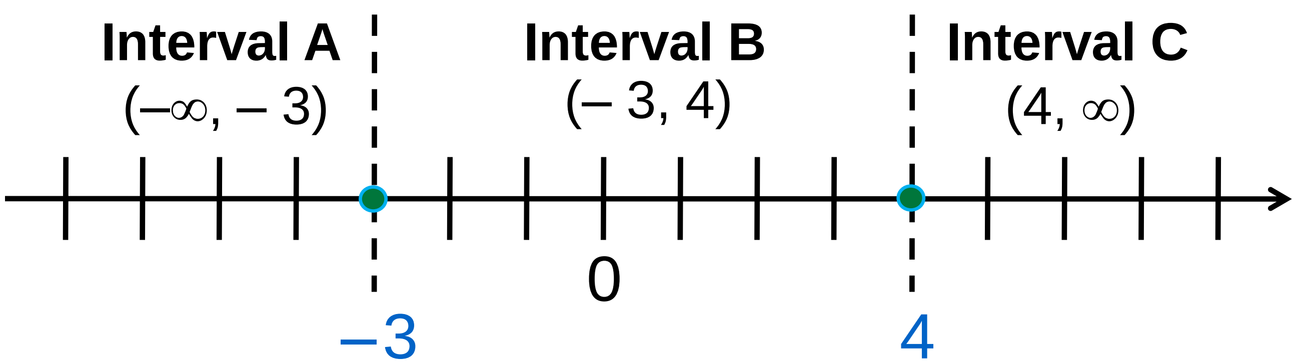
Step 1 Find the values of x that satisfy the corresponding quadratic *equation*.

$x^2 - x - 12 = 0$	Corresponding quadratic equation.
$(x+3)(x-4) = 0$	Factor.
$x + 3 = 0$ or $x - 4 = 0$	Zero-factor property.
$x = -3$ or $x = 4$	Solve each equation.

Example 4
Solving a Quadratic Inequality

Step 2 The two numbers –3 and 4 divide the number line into three intervals.

The expression $x^2 - x - 12$ will take on a value that is either ***less than*** zero or ***greater than*** zero on each of these intervals.



Example 4

Solving a Quadratic Inequality

Step 3 Choose a test value from each interval.

INTERVAL	TEST VALUE	Is $x^2 - x - 12 < 0$ True or False ?
A: $(-\infty, -3)$	-4	$(-4)^2 - (-4) - 12 < 0$? $8 < 0$ False
B: $(-3, 4)$	0	$0^2 - 0 - 12 < 0$? $-12 < 0$ True
C: $(4, \infty)$	5	$5^2 - 5 - 12 < 0$? $8 < 0$ False

Since the values in Interval B make the inequality true …

The solution set is $(-3, 4)$

Example 5

Solving a Quadratic Inequality

5(a) Solve: $2x^2 + 5x - 12 \geq 0$

Solution:

Step 1 Find the values of x that satisfy the corresponding quadratic *equation*.

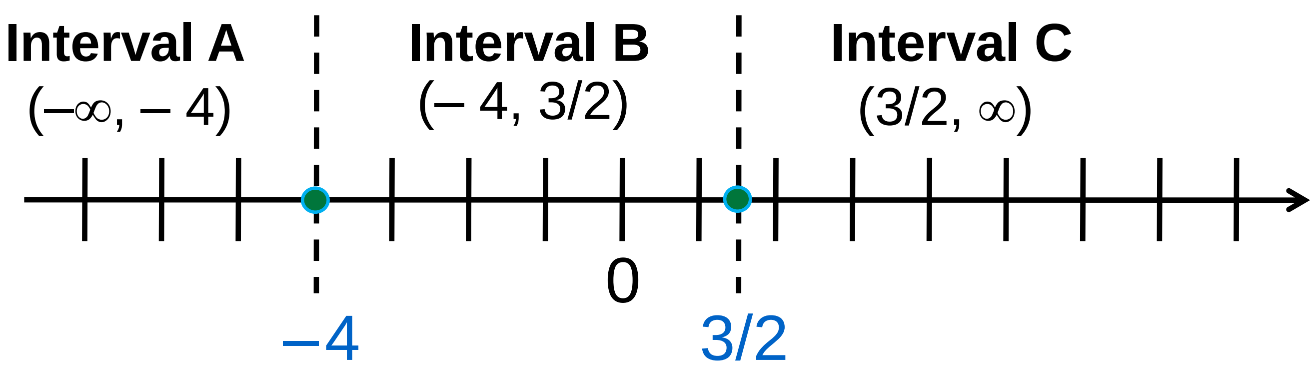
$2x^2 + 5x - 12 = 0$	Corresponding quadratic equation.
$(2x - 3)(x + 4) = 0$	Factor.
$2x - 3 = 0$ or $x + 4 = 0$	Zero-factor property.
$x = \frac{3}{2}$ or $x = -4$	Solve each equation.

Example 5

Solving a Quadratic Inequality

Step 2 The two numbers –4 and 3/2 divide the number line into three intervals.

The expression $2x^2 + 5x - 12$ will take on a value that is either ***less than*** zero or ***greater than*** zero on each of these intervals.



Example 5

Solving a Quadratic Inequality

Step 3 Choose a test value from each interval.

INTERVAL	TEST VALUE	Is $2x^2 + 5x - 12 \geq 0$ True or False ?
A: $(-\infty, -4)$	-5	$2(-5)^2 + 5(-5) - 12 \geq 0$? $13 \geq 0$ True
B: $(-4, 3/2)$	0	$2(0)^2 + 5(0) - 12 \geq 0$? $-12 \geq 0$ False
C: $(3/2, \infty)$	2	$2(2)^2 - (2) - 12 \geq 0$? $8 \geq 0$ True

Since the values in Intervals A and C make the inequality true …

The solution set is the union of intervals $(-\infty, -4) \cup \left[\frac{3}{2}, \infty\right)$

STRICT and NONSTRICT INEQUALITIES

Note

Inequalities that use the symbols $<$ and $>$ are ***strict inequalities***; $\leq$ and $\geq$ are used in ***nonstrict inequalities*.**

In **Example 4** the solutions of the equation were ***not*** included in the solution set since the inequality was a ***strict*** inequality.

In **Example 5**, the solutions of the equation ***were*** included in the solution set because of the ***nonstrict*** inequality

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Rational Inequalities

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RATIONAL INEQUALITIES

RATIONAL INEQUALITIES

A ***rational inequality***
is an inequality that has a rational expression for one or more terms.

To solve a rational equation, multiply each side by
the least common denominator (LCD) of the terms of the equation
and then solve the resulting equation.

Because a rational expression is not defined when its denominator is 0,
proposed solutions for which any denominator equals 0 are excluded from the solution set.

Solving a Rational Inequality

Solving a Rational Inequality

Step 1 Rewrite the inequality, if necessary, so that 0 is on one side and there is a single fraction on the other side.

Step 2 Determine the values that will cause either the numerator or the denominator of the rational expression to equal 0. These values determine the intervals of the number line to consider.

Step 3 Use a test value from each interval to determine which intervals form the solution set.

RATIONAL INEQUALITIES

Note

A value causing the *denominator* to equal zero
will *never be included* in the solution set.

If the inequality is *strict*,
any value causing the *numerator* to equal zero will be *excluded*.

If the inequality is *nonstrict*,
any value causing the *numerator* to equal zero will be *included*.

RATIONAL INEQUALITIES

Caution

Solving a rational inequality such as …

$$\frac{5}{x+4} \geq 1$$

by multiplying each side by $x+4$ to obtain $5 \geq x+4$ requires considering *two cases*, since the sign of $x+4$ depends on the value of x. If $x+4$ is negative, then the inequality symbol must be reversed.

The procedure used in the next two examples eliminates the need for considering separate cases.

Example 6

Solving a Rational Inequality

6(a) Solve: $$\frac{5}{x+4} \geq 1$$

Solution:

Step 1

$$\frac{5}{x+4} - 1 \geq 0$$

Subtract 1 so that 0 is on one side.

$$\frac{5}{x+4} - \frac{x+4}{x+4} \geq 0$$

Use $x+4$ as the common denominator.

$$\frac{5-(x+4)}{x+4} \geq 0$$

Write as a single fraction.

Note the careful use of parentheses.

$$\frac{1-x}{x+4} \geq 0$$

Combine terms in the numerator, being careful with signs.

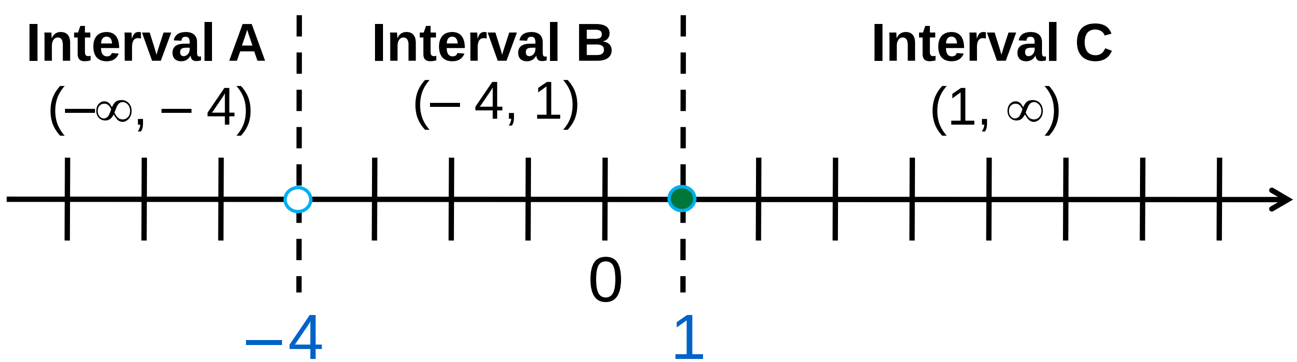
Example 6

Solving a Rational Inequality

Step 2 The quotient possibly changes sign only where x-values make the numerator or denominator 0. This occurs at …

$$1 - x = 0 \quad \text{or} \quad x + 4 = 0$$

$$x = 1 \quad \text{or} \quad x = -4$$



Example 6

Solving a Rational Inequality

Step 3 Choose a test value from each interval.

INTERVAL	TEST VALUE	Is $\dfrac{5}{x+4} \geq 1$ True or False ?
A: $(-\infty, -4)$	-5	$\dfrac{5}{-5+4} \geq 1$? $\quad -5 \geq 1$ False
B: $(-4, 1)$	0	$\dfrac{5}{0+4} \geq 1$? $\quad \dfrac{5}{4} \geq 1$ True
C: $(1, \infty)$	2	$\dfrac{5}{2+4} \geq 1$? $\quad \dfrac{5}{6} \geq 1$ False

The values in the interval (4, 1) satisfy the original inequality.

The value 1 makes the nonstrict inequality true, so it must be *included*.

The value –4 makes the denominator 0, so it must be *excluded*.

The solution set is $(-4, 1]$

RATIONAL INEQUALITIES

Caution

***Be careful with the endpoints of the intervals when solving rational inequalities*.**

Example 7

Solving a Rational Inequality

7(a) Solve: $$\frac{2x-1}{3x+4} < 5$$

Solution:

$$\frac{2x-1}{3x+4} - 5 < 0$$ Subtract 5.

$$\frac{2x-1}{3x+4} - \frac{5(3x+4)}{3x+4} < 0$$ Common denominator is $3x+4$.

$$\frac{2x-1-5(3x+4)}{3x+4} < 0$$ Write as a single fraction.

Example 7

Solving a Rational Inequality

Solution (cont'd):

$$\frac{2x - 1 - 15x - 20}{3x + 4} < 0$$

Distributive property.

Be careful with signs.

$$\frac{-13x - 21}{3x + 4} < 0$$

Combine like terms in the numerator.

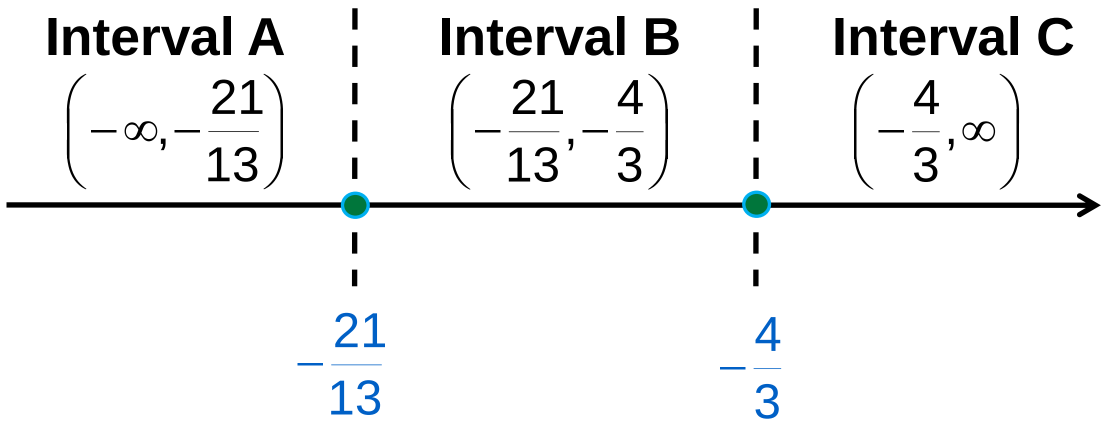
Set the numerator and denominator equal to 0 and solve the resulting equations to find the values of *x* where sign changes may occur.

$$-13x - 21 = 0 \quad \text{or} \quad 3x + 4 = 0$$

$$x = -\frac{21}{13} \quad \text{or} \quad x = -\frac{4}{3}$$

Example 7

Solving a Rational Inequality



Now choose test values from the intervals and verify that …

– 2 from Interval A makes the inequality true;

– 1.5 from Interval B makes the inequality false;

0 from Interval C makes the inequality true.

Because of the $<$ symbol, neither endpoint satisfies the inequality …

The solution set is $\left(-\infty, -\frac{21}{13}\right) \cup \left(-\frac{4}{3}, \infty\right)$